Probability

Distributions in $\mathbb{R}^2$

Exercice 1 Let X and Y de two independent random variables such that $X \hookrightarrow \Gamma(1, a)$ and $Y \hookrightarrow \Gamma(1, b)$ where $(a, b) \in \mathbb{N}^*$.

a) Find the distribution of the pair $(S = X + Y, Z = \frac{X}{X+Y})$.

b) Find the distributions of S and Z. Are S and Z independent?

Exercice 2 Let X and Y de two independent random variables such that $X \hookrightarrow \mathcal{N}(m, \sigma^2)$ and $Y \hookrightarrow \mathcal{N}(\tilde{m}, \tilde{\sigma}^2)$.

Find the distribution of $X + Y$.

Exercice 3 Let (X, Y) be a pair of random variables defined on a probability space $(\Omega, \mathcal{A}, P)$ and with values in $\mathbb{N} \times \mathbb{R}$ such that $\forall k \in \mathbb{N}$, $\forall B \in Bor(\mathbb{R})$,

$$P(X = k \cap Y \in B) = \int_{B \cap]0,+\infty[} e^{-y} \frac{y^k \theta^p}{k!(p-1)!} y^{p-1} e^{-\theta y} dy$$

with $p \in \mathbb{N}^*$ and $\theta > 0$.

Find the marginal distributions of X and Y.

Exercice 4 Let $(X_n)_{n \in \mathbb{N}^*}$ be a sequence of i.i.d random variables such that $P(X_n = -1) = 1 - p$ and $P(X_n = 1) = p$ with $p \in]0, 1[$. For $n \geq 2$, we define $Y_n = X_{n-1} X_n$.

a) Find the distributions of (Y_n) and of (Y_n, Y_{n+1}).

b) Determine $E[Y_{n+1} | Y_n]$ and find its distribution.