Variance reduction methods

Christophe Chorro

Homework 2 (For July 21 (In the morning...))

Computer programs have to be joined and may be written in the language you want (R, Matlab, Gauss, VBA...).

Let G be a $\mathcal{N}(0,1)$. We want to compare several variance reduction methods to compute for $t \geq 0$

$$I = \mathbb{E}[1_{\{G>0\}} e^{tG}].$$

In the sequel, gaussian random variables have to be generated using the **Box Muller method.**

The following computations will be done for $t \in \{0,1,2,3\}$.

1] Compute by classical Monte Carlo simulations ($N = 10000$) I and give the corresponding confidence interval (level 95%).

2] Compute by Monte Carlo simulations with antithetic variables ($N = 10000$) I and give the corresponding confidence interval (level 95%).

Hint : G and $-G$ have the same distribution...

3] a) Prove that

$$I = e^{\frac{t^2}{2}} - \mathbb{E}[1_{\{G\leq 0\}} e^{tG}].$$

b) Use the preceding equality to compute by Monte Carlo simulations with control variate ($N = 10000$) I and give the corresponding confidence interval (level 95%).

4] a) Prove that

$$I = e^{\frac{t^2}{2}} \mathbb{E}[1_{\{G\leq t\}}].$$

b) Use the preceding equality to compute with a new Monte Carlo estimator ($N = 10000$) I and give the corresponding confidence interval (level 95%).

c) Explain why this estimator corresponds to an importance sampling method.

5] Compare the different results and give intuitive explanations.