Monte Carlo Method

Christophe Chorro

Homework 1 (For July 10)

Pricing Basket options by MC Methods

Computer programs have to be joined and may be written in the language you want (R, Matlab, Gauss, VBA...).

If you have some questions or problems you can join me at

christophe.chorro@gmail.com.

Let G_1 and G_2 be two independent $\mathcal{N}(0,1)$ and $(\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}) \in \mathbb{R}^4$. We define

$$S_1 = e^{\sigma_{11}G_1 + \sigma_{12}G_2}$$
$$S_2 = e^{\sigma_{21}G_1 + \sigma_{22}G_2}.$$

For $a_1, a_2 > 0$ we are interested in the numerical computation of

$$E\left[\left(K - \underbrace{(a_1 S_1 + a_2 S_2)}_{X}\right)_+\right]. \tag{1}$$

Part I : Suppose that $a_1 = a_2 = 0.5$, $K = 1$, $\sigma_{11} = \sigma_{22} = 0.1$ and $\sigma_{12} = \sigma_{21} = 0.15$.

a) Using the Box Muller method to generate i.i.d $\mathcal{N}(0,1)$, compute by classical Monte Carlo simulations ($N = 10000$) (1) and give the corresponding confidence interval (level 95%).

Hint : The variance of $\left(K - \underbrace{(a_1 S_1 + a_2 S_2)}_{X}\right)_+$ may be approximated using the empirical variance (Lecture 1).

b) Do the same thing using Gauss Marsiglia method to generate i.i.d $\mathcal{N}(0,1)$.

Part II : Control Variate Method

We suppose that $(\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22})$ are small,

a) When $(\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22})$ are small, show that

$$Log\left(\frac{a_1 S_1 + a_2 S_2}{a_1 + a_2}\right) \sim Y = \frac{a_1}{a_1 + a_2} Log(S_1) + \frac{a_2}{a_1 + a_2} Log(S_2).$$

b) Show that $X \sim (a_1 + a_2) e^{\frac{Y}{a_1 + a_2}}$ where

$$Y \hookrightarrow \mathcal{N}\left(0, (a_1\sigma_{11} + a_2\sigma_{21})^2 + (a_1\sigma_{12} + a_2\sigma_{22})^2\right).$$

c) Compute explicitly $\mathbb{C}$ where

$$\mathbb{C} = E\left[\left(K - (a_1 + a_2) e^{\frac{Y}{a_1 + a_2}}\right)_+\right]. \tag{2}$$

Hint : Remind the proof of the Black and Scholes formula.

Suppose that $a_1 = a_2 = 0.5$, $K = 1$, $\sigma_{11} = \sigma_{22} = 0.1$ and $\sigma_{12} = \sigma_{21} = 0.15$.

d) Remarking that

$$E\left[(K - X)_+\right] = E\left[(K - X)_+ - (K - Y)_+\right] + \mathbb{C},$$

use a classical Monte Carlo Method ($N = 10000$) to approximate

$$E\left[(K - X)_+ - (K - Y)_+\right].$$

What is the corresponding confidence interval (level 95%) for $E\left[(K - X)_+\right]$.

e) Compare with the results of part I and give an intuitive explanation of the results.